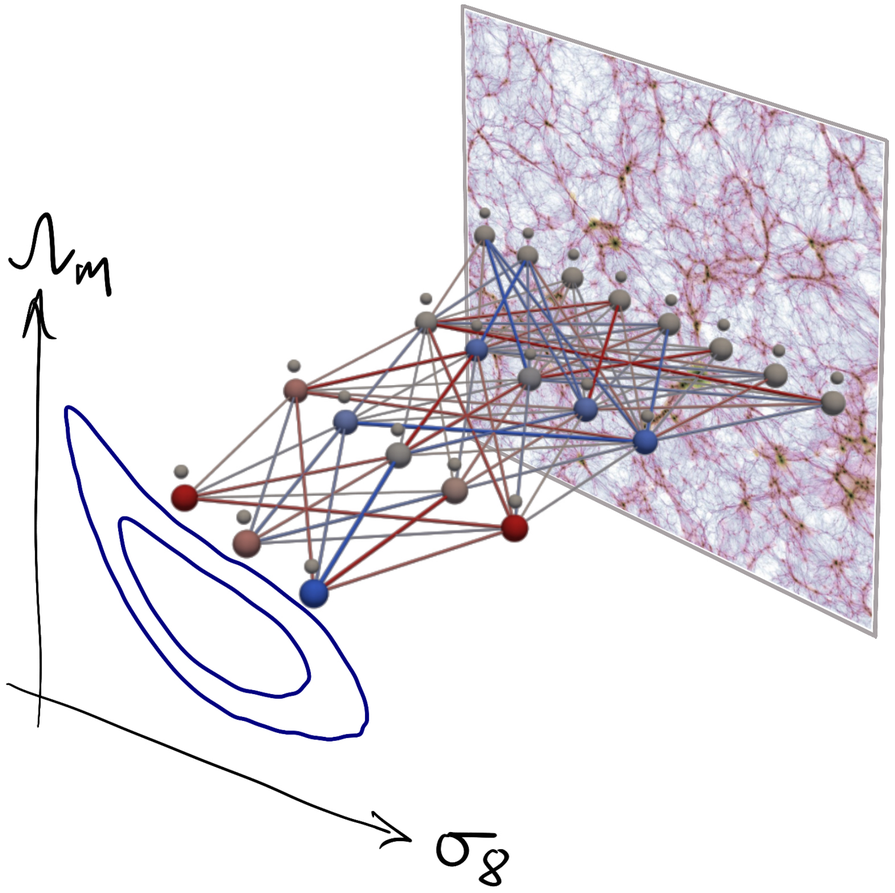
What is Simulation-Based Inference?



Why simulation-based inference (SBI)?

$$p(\pi \mid \hat{x}) \propto p(\hat{x} \mid x[\pi], \pi)\, p(\pi)$$

- **Likelihood** $p(\hat{x} \mid \pi)$ may be non-Gaussian
- **Expectation** $x[\pi]$ is a complex non-linear function of π
- **Covariance** Σ may depend on π
- n_x **is increasing** with new surveys, inversion of $\hat{\Sigma}$

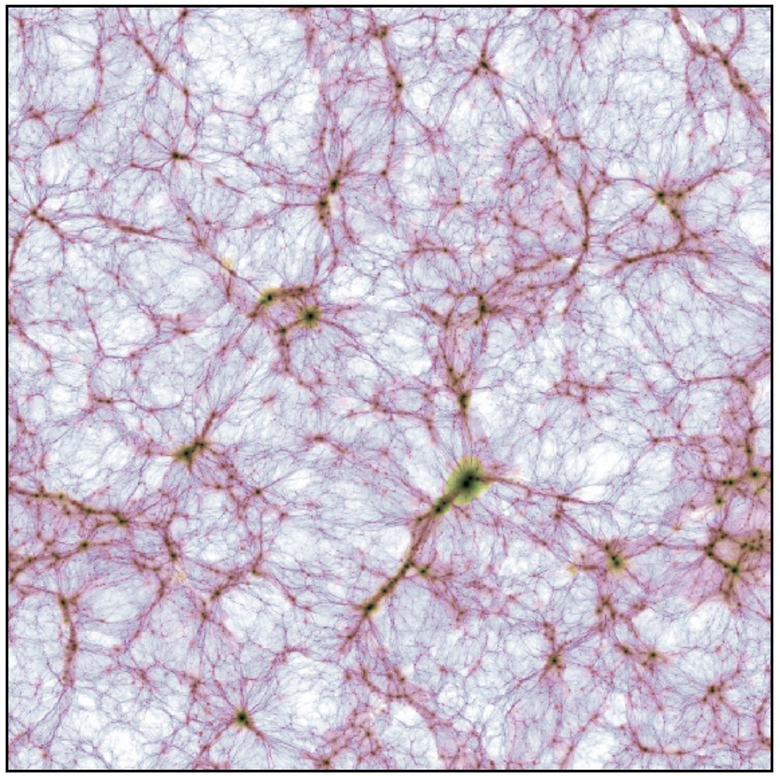
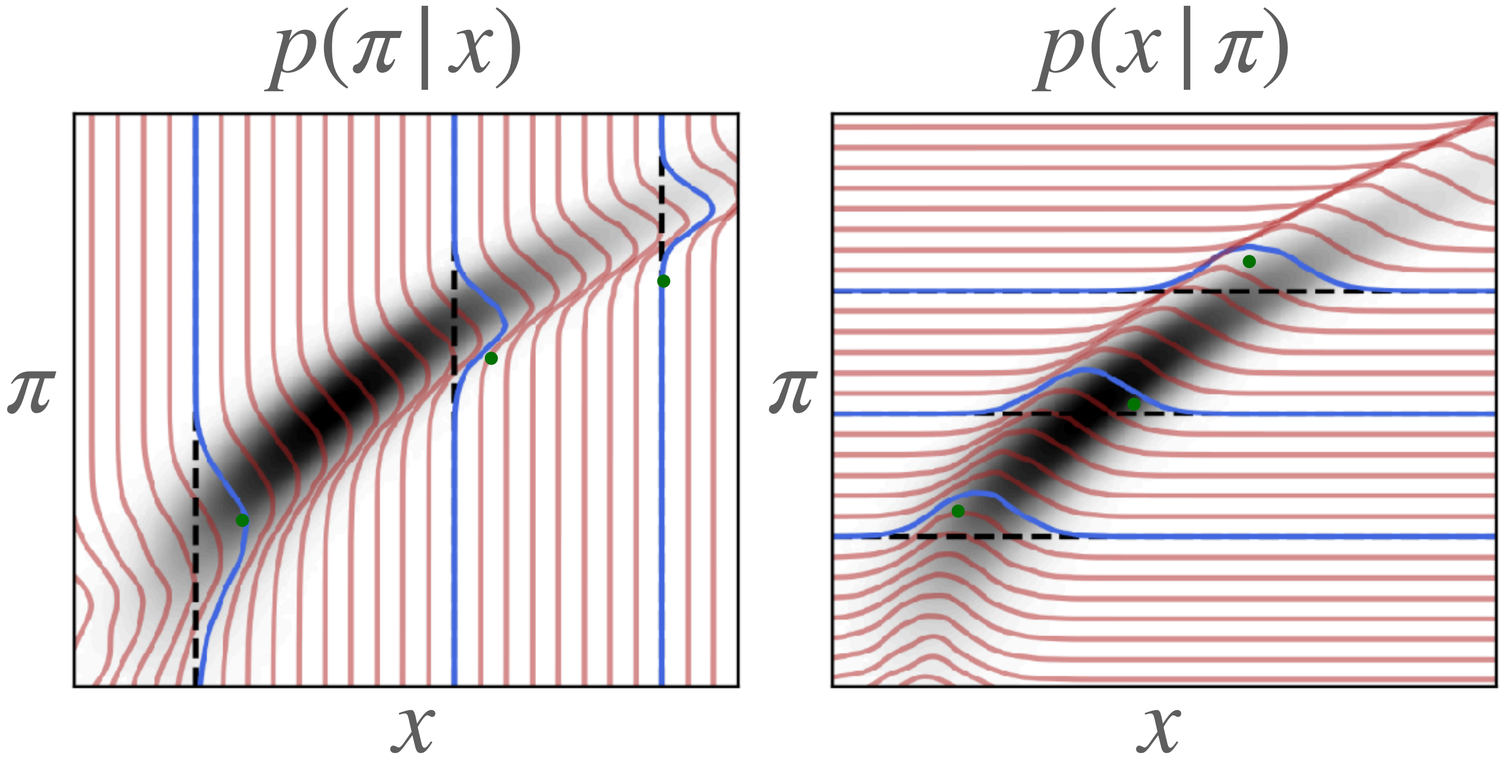


Image: IllustrisTNG

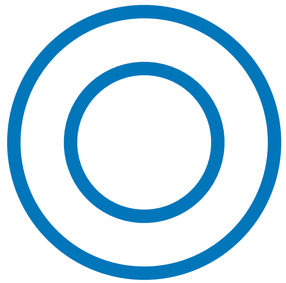
Simulation-based inference (SBI)



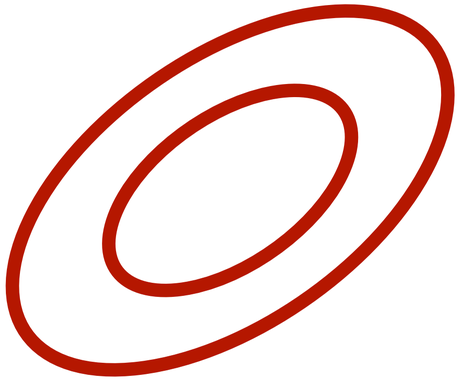
- **Analytic methods**: model PDF $p(x \mid \pi)$ across space (x, π)

- **SBI**: fit the expectation $x[\pi]$, covariance Σ and PDF $p(x \mid \pi)$ from simulations (x, π)

How do we implement SBI? Normalising flows

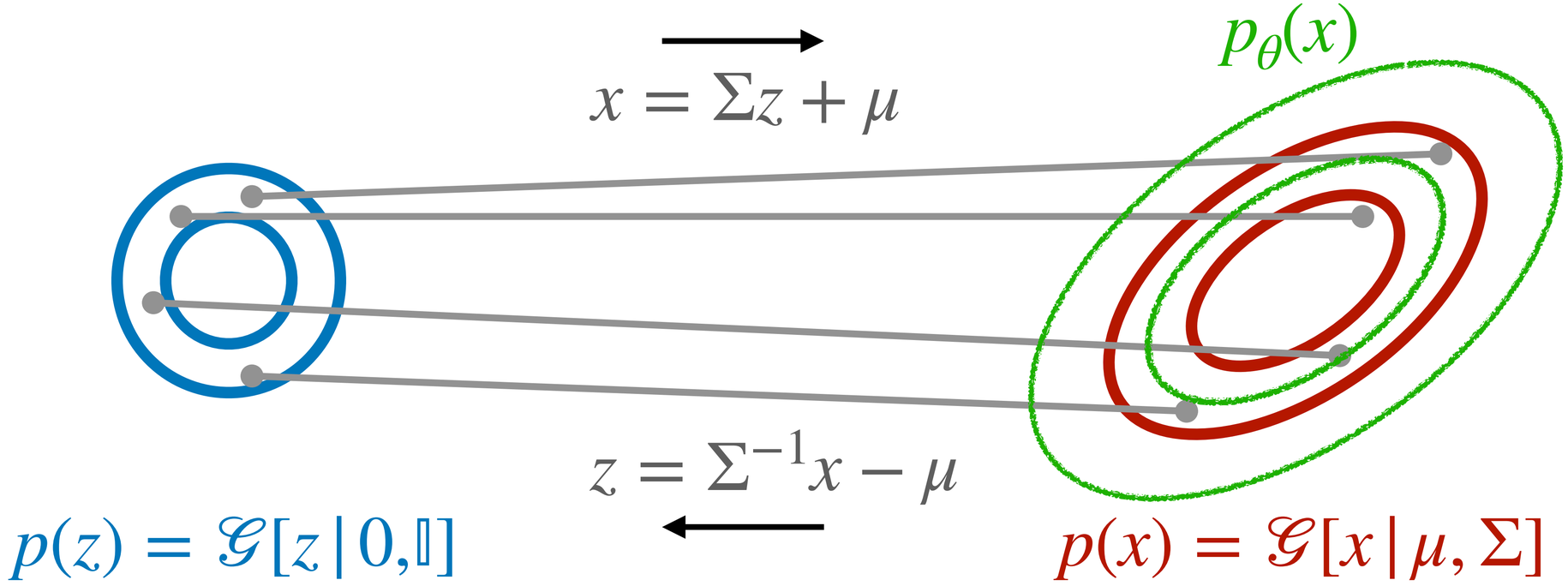


$$p(z) = \mathcal{G}[z\,|\,0,\mathbb{I}]$$



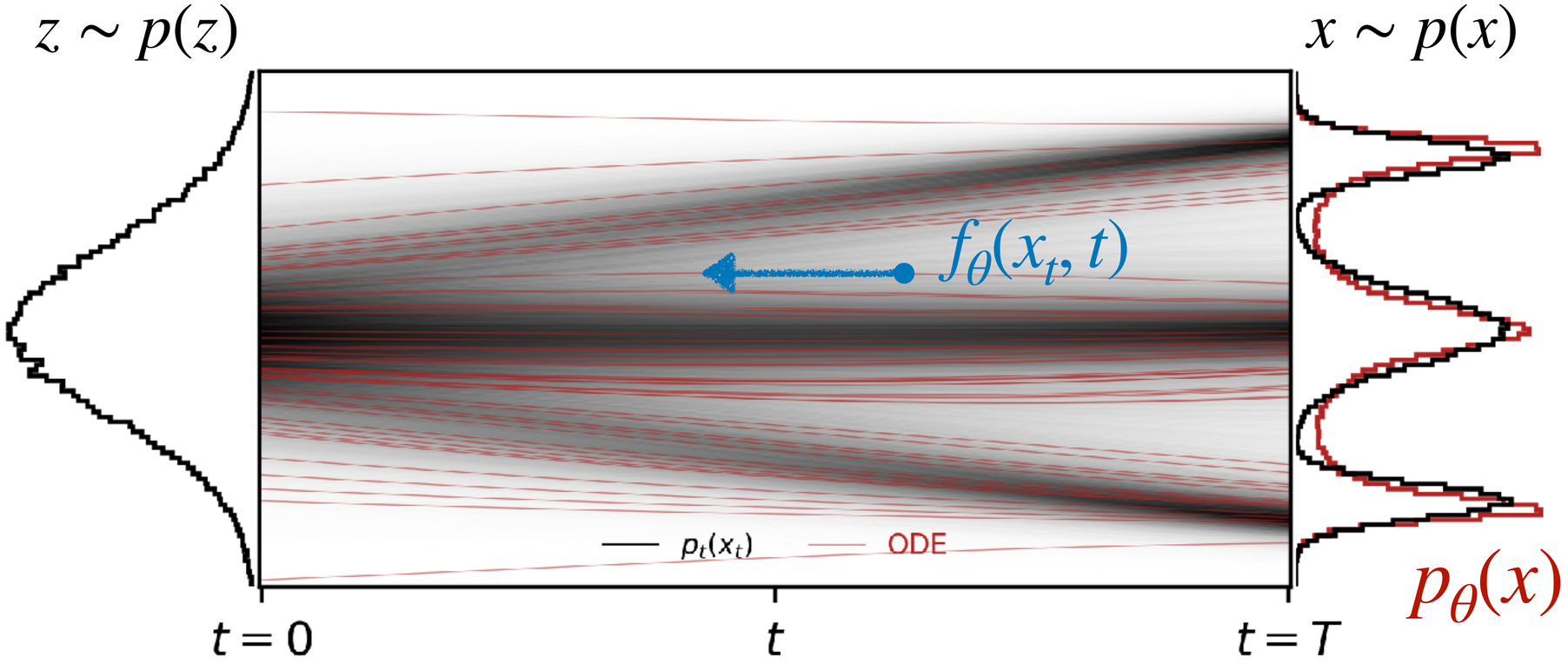
$$p(x) = \mathcal{G}[x\,|\,\mu,\Sigma]$$

Normalising flows: Analytic example



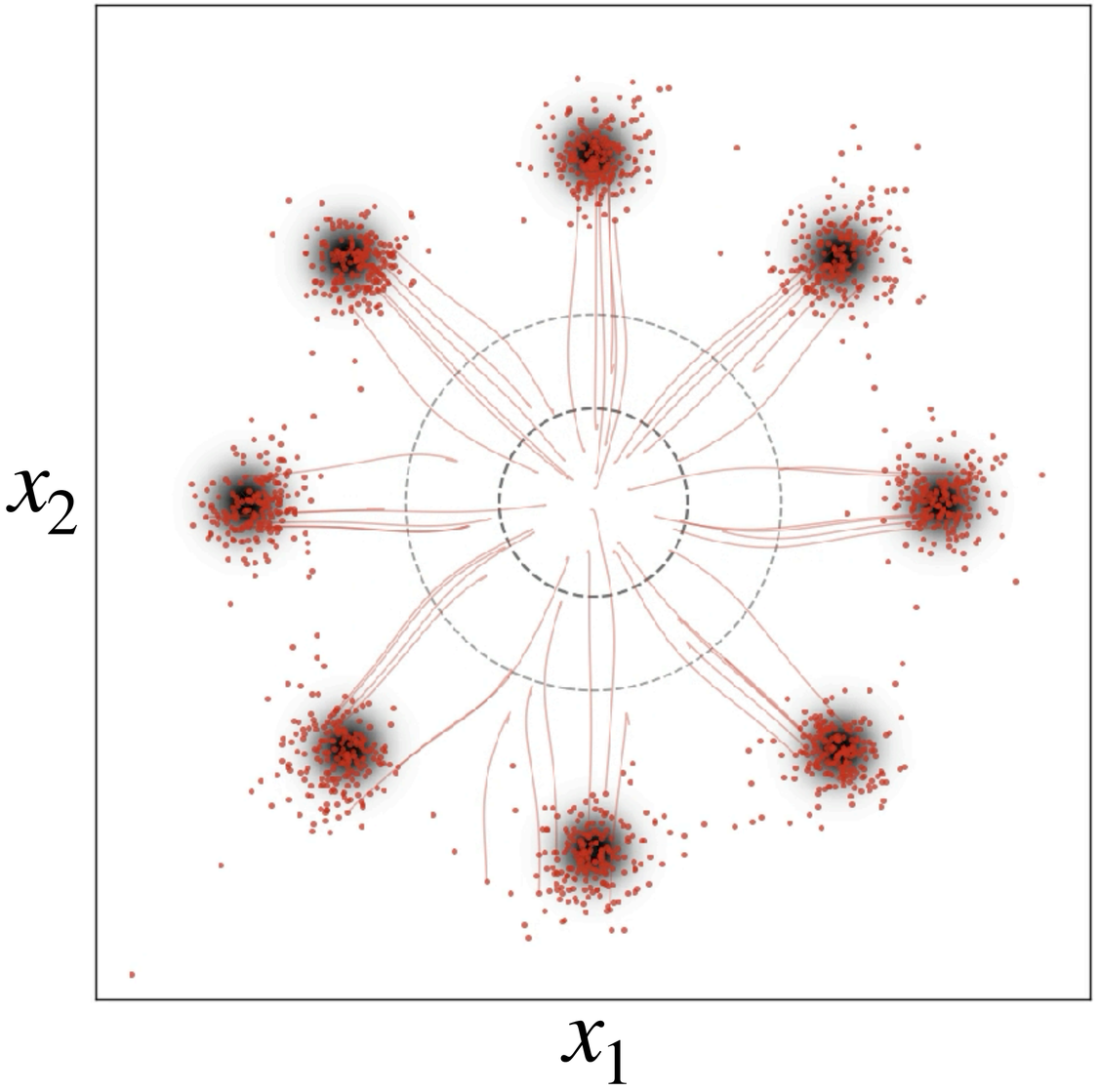
$$\log p_\theta(x) = \log p(z) + \text{Jacobian term}$$

State-of-the-art: Continuous normalising flows



$$\log p_\theta(x) = \log p(z) + \int_{t=0}^{t=T} \mathrm{d}t \, \nabla_{x_t} \cdot f_\theta(x_t, t)$$

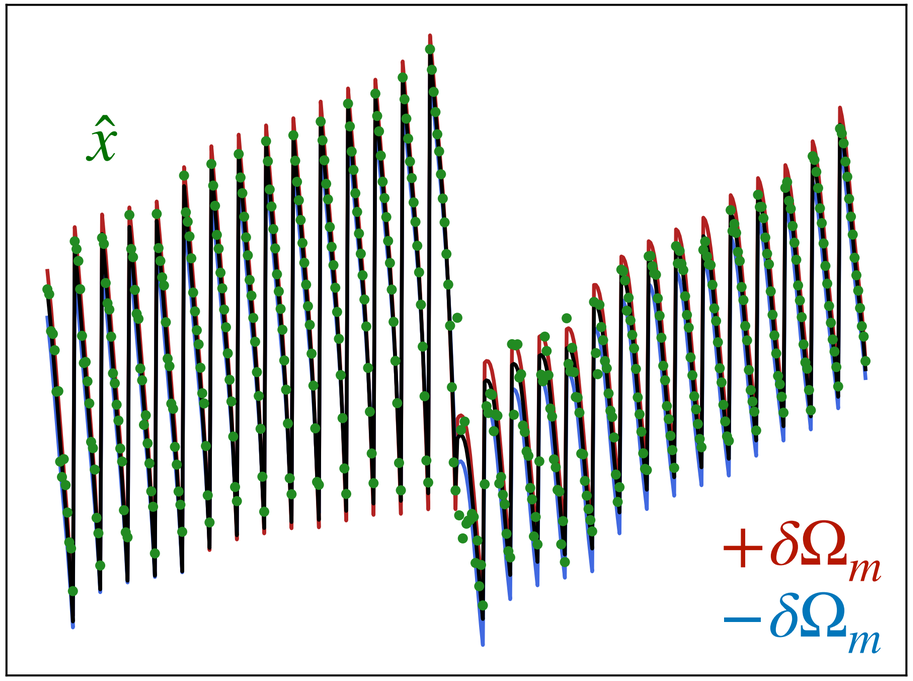
Continuous normalising flows



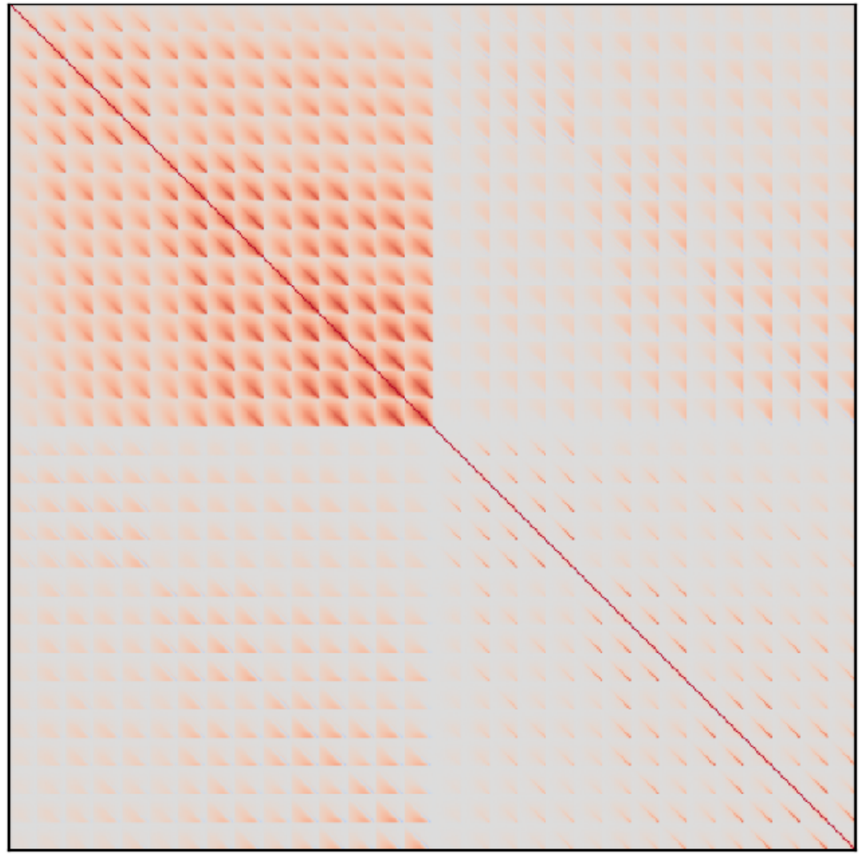
Simple but relevant case: Gaussian linear models

$$\hat{x} \sim \mathcal{G}\left[\hat{x} \,|\, x[\pi], \Sigma\right]$$

$x[\pi]$



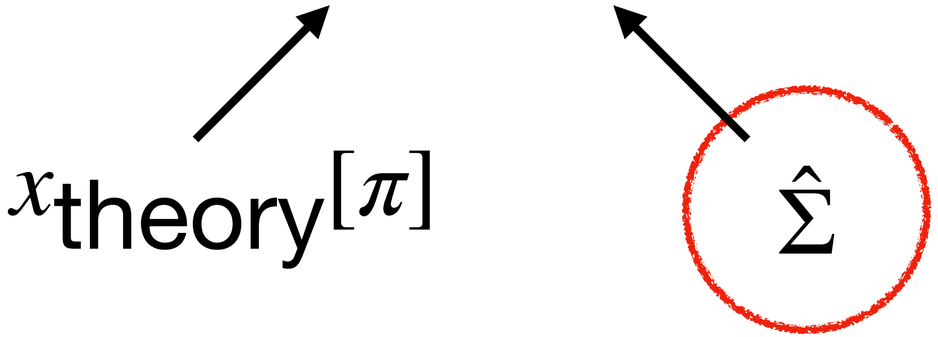
Σ



"Classical" likelihood inference

$$\hat{x} \sim \mathcal{G}\left[\hat{x} \,|\, x[\pi], \Sigma\right]$$

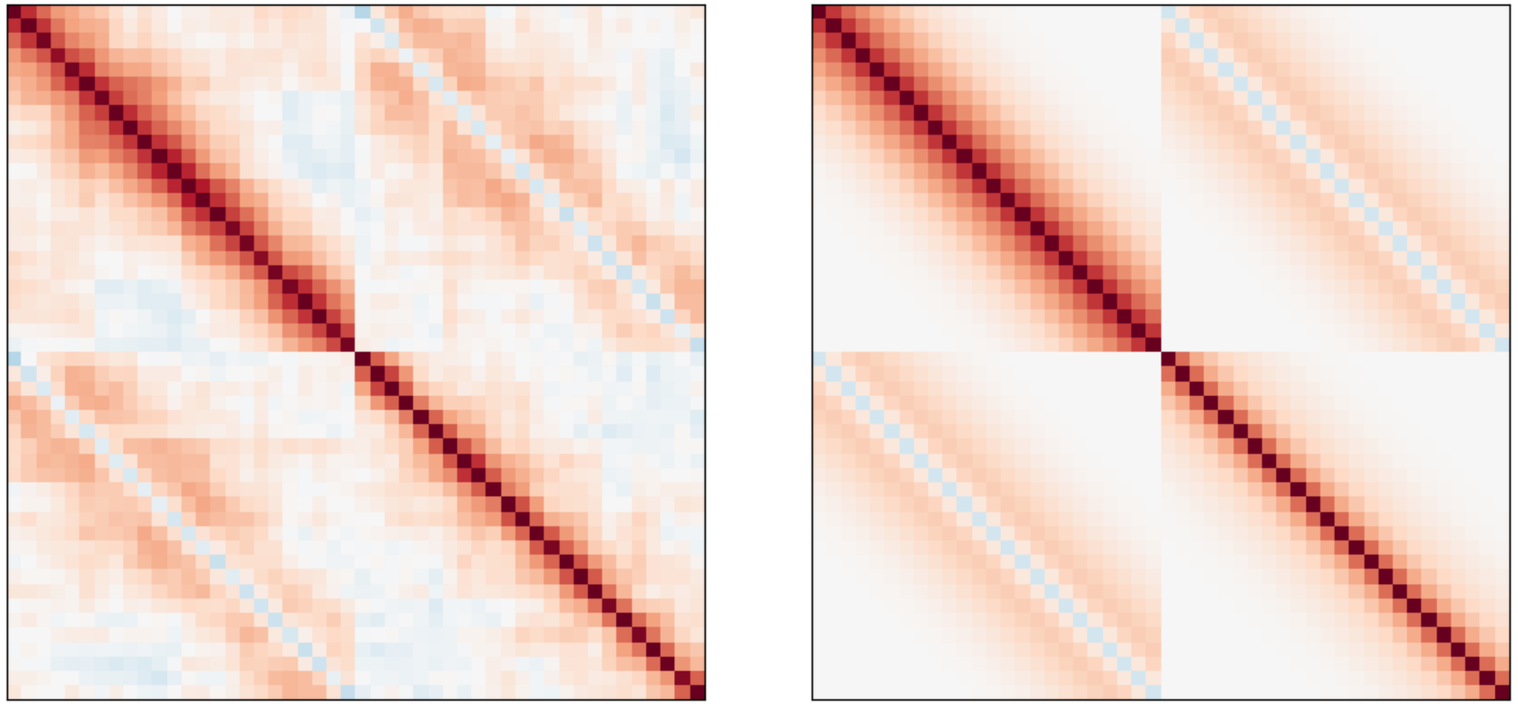
$$\mathcal{G}\left[\hat{x} \,|\, x_{\text{theory}}[\pi], \hat{\Sigma}\right]$$



Covariance matrix estimation

$$\hat{x} \sim \mathcal{G}\left[\hat{x} \,|\, x[\pi], \Sigma\right]$$

$\hat{\Sigma}$ $n_s = 285$ Σ

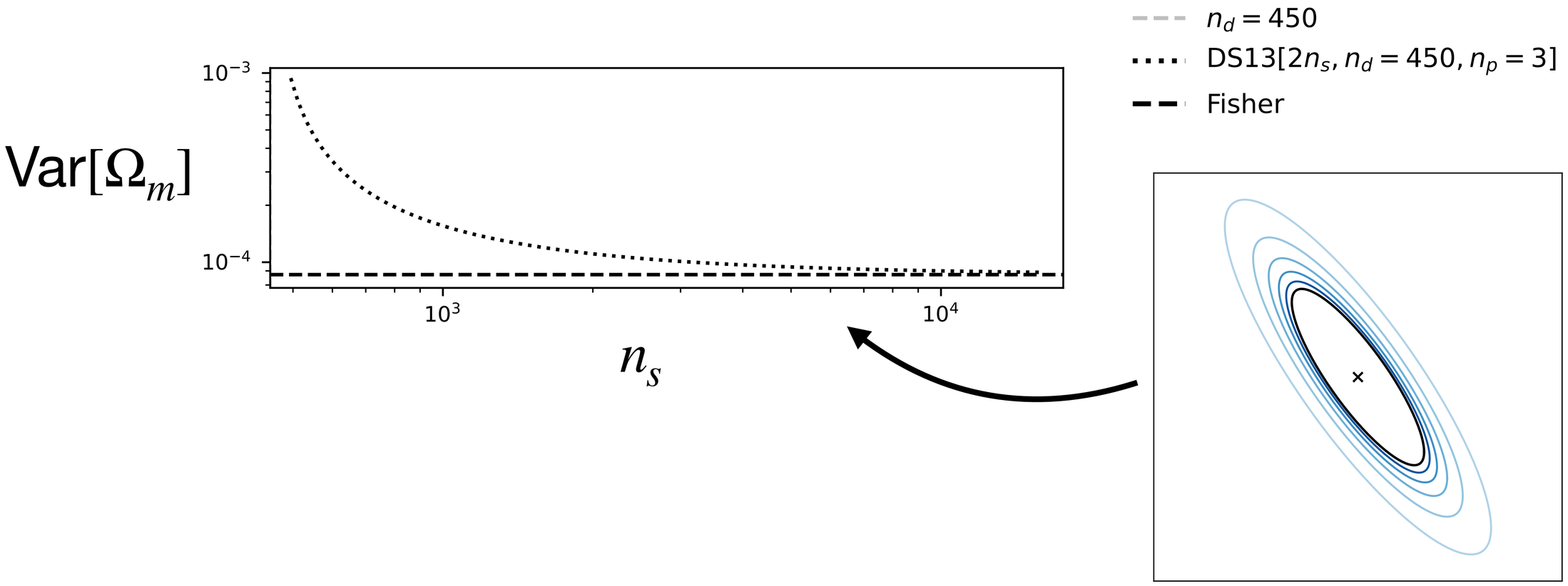


SBI

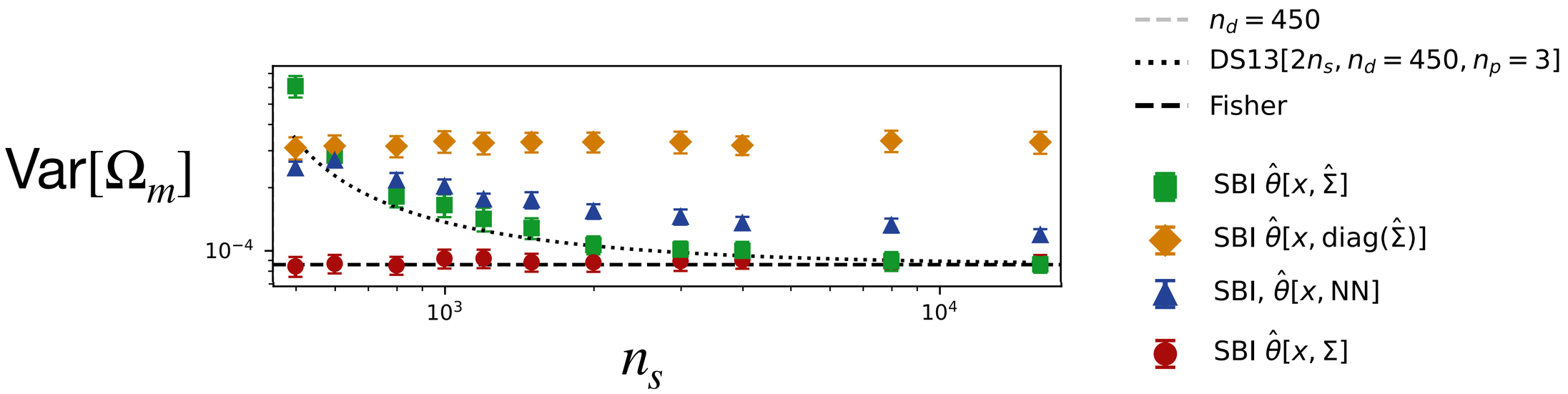
$$p(\pi \mid \hat{x}) \propto \mathcal{G}\left[\hat{x} \mid x[\pi], \Sigma\right] p(\pi)$$

$$p(\pi \mid \hat{\pi}) \propto \quad p(\hat{\pi} \mid \pi, \phi) \quad p(\pi)$$

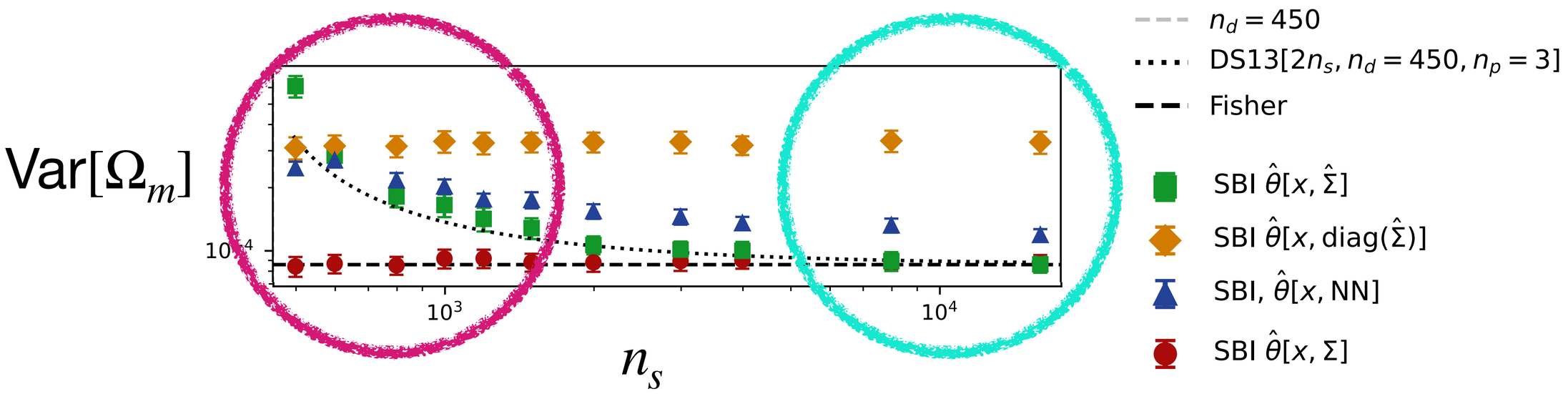
Posterior widths: SBI vs. Gaussian likelihood analysis



Posterior widths: SBI vs. Gaussian likelihood analysis



Posterior widths: SBI vs. Gaussian likelihood analysis



Cosmological measurements present a rising tide...

SBI can fit a likelihood PDF with no assumptions on data generating process....